QUALITATIVE METHODS IN KPZ UNIVERSALITY

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Abstracts

Tom Alberts

Geometric Methods for Last Passage Percolation

Abstract: In an attempt to generalize beyond solvable methods of analysis for last passage percolation, recently Eric Cator (Radboud University, Nijmegen) and I have started analyzing the piecewise linearity of the last passage model. The tools we use to this point are purely geometric, but have the potential advantage that they can be used for very general choices of random inputs. I will describe the very pretty geometry of the last passage model, our work in progress to use it to produce probabilistic information, and some connections to algebraic geometry.

Amol Aggarwal

Current Fluctuations of the Stationary ASEP and Six-Vertex Model

Abstract: We consider the following three models from statistical mechanics: the asymmetric simple exclusion process, the stochastic six-vertex model, and the ferroelectric symmetric six-vertex model. It had been predicted by the physics communities for some time that the limiting behavior for these models, run under certain classes of translation-invariant (stationary) boundary data, are governed by the large-time statistics of the stationary Kardar-Parisi-Zhang (KPZ) equation. The purpose of this talk is to explain these predictions in more detail and survey some of our recent work that verifies them.

Guillaume Barraquand

ASEP on the positive integers with an open boundary and the KPZ equation in a half space

Abstract: We consider the ASEP on the positive integers with an open boundary at the origin, and prove a Tracy-Widom GOE limit theorem for the current at the origin when the boundary parameters are chosen so as to enforce an average density $1/2$ near the origin. We study half-line ASEP through a half-space stochastic six-vertex model, the main technical tool being a half-space variant of Macdonald processes that generalize Pfaffian Schur processes. We will also discuss the weakly asymmetric limit of ASEP height function to the KPZ equation in a half-space with Neumann boundary condition, and we expect the Laplace transform of the solution to the KPZ equation to be related to a multiplicative functional of the GOE point process, the functional being slightly different than in the analogous full-space result.

Based on joint work with Alexei Borodin, Ivan Corwin and Michael Wheeler.

Jinho Baik

Multi-time distribution of periodic TASEP

Abstract: We consider periodic TASEP with periodic step initial condition, and evaluate the joint distribution of the locations of m particles. For arbitrary indices and times, we find a formula for the multi-time, multi-space joint distribution in terms of an integral of a Fredholm determinant. We then discuss the large time limit in the so-called relaxation scale. The one-point distributions for other initial conditions are also going to discussed.

Based on joint work with Zhipeng Liu (NYU).

Alexander Bufetov

On the Gibbs property for determinantal point processes

Abstract: The talk will describe conditional measures in a bounded interval, with respect to fixing the configuration in the exterior, for one-dimensional determinantal point processes governed by integrable kernels. These conditional measures are given by orthogonal polynomial ensembles with explicitly computed weights. Examples include classical determinantal point processes of random matrix theory such as the sine-process or the process governed by the Bessel kernel.

Based on the preprint arXiv:1605.01400.

Francis Comets

Mean-field directed polymers on a complete graph.

Abstract: Consider discrete directed polymers on a finite complete graph of size N (instead of a lattice). The model can be formulated as a product of i.i.d. NxN random matrices, and large time asymptotics is captured by Lyapunov exponents and Furstenberg measure. I will detail this correspondence. For some specific choice of the disorder variables, the model is exactly solvable: the polymer height evolves like a random walk (yielding the mean height) with N "decorations" obeying an explicit law. In this case, asymptotics of large size N can further be taken.

Based on joint work with G. Moreno and A. Ramirez.

Evgeni Dimitrov

The ASEP and Hall-Littlewood Gibbsian line ensembles

Abstract: We consider the ASEP started from step initial condition and investigate the large time T distribution of the height function. Conjecturally, under $T^{2/3}$ spatial and $T^{1/3}$ fluctuation scaling (also known as KPZ scaling) the asymptotic behavior is described by the Airy_2 process. We provide further evidence for this conjecture by showing that under the KPZ scaling the height function is tight in the space of continuous curves. In the first part of the talk we will discuss the connection between the ASEP and the ascending Hall-Littlewood process, which was recently discovered by Borodin-Bufetov-Wheeler. The second part of the talk explains how to realize ASEP as the top curve of a line ensemble with a Gibbsian resampling property and how one point-tightness of the top curve can be propagated to the tightness of the entire curve.

Based on joint work with Ivan Corwin.

Dirk Erhard

Discretisation of regularity structures

Abstract: The theory of regularity structures is a framework developed by Martin Hairer that allows to renormalise stochastic PDEs that are ill-posed in the classical sense. The theory is very successful when applied to approximations of the equation at hand that are defined on $\mathbb{R}^{\wedge}d$. However, it presently does not in general apply to discrete approximation schemes. In this talk I will describe a set-up that circumvents this problem.

Based on joint work with Martin Hairer.

Patrik Ferrari

Universality of the GOE Tracy-Widom distribution for TASEP with arbitrary particle density

Abstract: We consider TASEP in continuous time with non-random initial conditions and arbitrary fixed density of particles rho. We show GOE Tracy-Widom universality of the one-point fluctuations of the associated height function. The result phrased in last passage percolation language is the universality for the point-to-line problem where the line has an arbitrary slope.

Based on joint work with Alessandra Occelli.

Vadim Gorin

Tilings and non-intersecting paths beyond integrable cases

Abstract: The talk is about a class of systems of 2d statistical mechanics, such as random tilings, noncolliding walks, log-gases and random matrix-type distributions. Specific members in this class are integrable, which means that available exact formulas allow delicate asymptotic analysis leading to the Gaussian Free Field, sine-process, Tracy-Widom distributions. Extending the results beyond the integrable cases is challenging. I will speak about a recent progress in this direction: about universal local limit theorems for a class of lozenge and domino tilings, noncolliding random walks; and about GFF-type asymptotic theorems for global fluctuations in these systems and in discrete beta log–gases.

Dmitry Ioffe

Low temperature interfaces and level lines in the critical prewetting regime

Abstract: Complete wetting in the context of the low temperature two-dimensional Ising model is characterized by creation of a mesoscopic size layer of the "-" phase above an active substrate. Adding a small positive magnetic field h makes "-"-phase unstable, and the layer becomes only microscopically thick. Critical prewetting corresponds to a continuous divergence of this layer as h tends to zero. There is a conjectured 1/3 (diffusive) scaling leading to Ferrari-Spohn diffusions. Rigorous results were established for polymer models of random and self-avoiding walks under vanishing area tilts.

A similar 1/3-scaling is conjectured to hold for top level lines of low temperature SOS-type interfaces in three dimensions. In the latter case, the effective local structure is that of ordered walks, again under area tilts. The conjectured scaling limits (rigorously established in the random walk context) are ordered diffusions driven by Airy Slatter determinants.

Based on joint walks with Senya Shlosman, Yvan Velenik and Vitali Wachtel.

Chris Janjigian

Large deviations for certain inhomogeneous corner growth models

Abstract: This talk considers the large deviation properties of a generalization of the classical corner growth model with exponential weights, in which the rates of the exponential are drawn randomly in an appropriate way. We will discuss some computations of rate functions in the quenched and annealed versions of the model, along with some interesting properties of large deviations in this model.

Based on joint work with Elnur Emrah.

Arjun Krishnan

Stationary random walks on the lattice

Abstract: We consider translation invariant measures on configurations of nearest-neighbor arrows on the integer lattice. Following the arrows from each point on the lattice produces a family of semi-infinite non-crossing walks. We classify the collective behavior of these walks under mild assumptions: they either coalesce almost surely or form bi-infinite trajectories. Bi-inifinite trajectories form measure-preserving dynamical systems, have a common asymptotic direction in 2d, and possess other nice entropic properties. We use our theory to classify the behavior of non-crossing semi-infinite geodesics in stationary first- and last-passage percolation. We also partially answer a question raised by C. Hoffman about the limiting empirical measure of weights seen by geodesics.

Based on joint work with Jon Chaika.

Konstantin Matetski

The KPZ fixed point

Abstract: We describe the distribution function of the KPZ fixed point, which is the Markov process at the centre of the KPZ universality class. To this end, we derive an explicit Fredholm determinant formula for the multipoint distribution of the height function of the totally asymmetric simple exclusion process with arbitrary initial condition and take its 1:2:3 scaling limit. The method is by solving the biorthogonal ensemble/non intersecting path representation found by Borodin, Ferrari, Praehofer and Sasamoto.

Based on the recent result by K.Matetski, J.Quastel and D.Remenik.

Hirofumi Osada

Interacting Brownian motions in infinite dimensions with logarithmic potentials and Airy point process

Abstract: I prove the N-particle system reversible with respect to the distribution of eigenvalues of GUE converge to the solution of the infinite-dimensional stochastic differential equation associated with the Airy point process with $\beta = 2$ under the soft-edge scaling. For the inverse temperature $\beta = 1, 4$, I show the limit points of the associated N-particle system is a solution of the infinite-dimensional stochastic differential equation associated with the Airy point process with $\beta = 1, 4$, respectively. I use general theories for the convergence of N-particle systems for interacting Brownian motions and pathwise uniqueness result of solutions for this kind of infinite dimensional stochastic differential equations.

Based on joint works with Hideki Tanemura (Chiba University), Yosuke Kawamoto, Shota Osada (Kyushu University).

Nicolas Perkowski

Martingale solutions to the KPZ equation

Abstract: Recently we obtained a probabilistic understanding of the KPZ equation and were able to prove that the corresponding stationary or near-stationary martingale problem is well posed. The martingale problem is a powerful tool for proving convergence to the KPZ equation, and in particular for establishing its (weak) universality. I will discuss the uniqueness result and how to use it to prove the convergence for various models.

Based on joint works with Joscha Diehl, Patricia Goncalves, Massimiliano Gubinelli and Marielle Simon.

Leonid Petrov
TASEP in continuous inhomogeneous space

Abstract: We discuss a version of TASEP in continuous time and space, in which the space can be equipped with arbitrary inhomogeneity. This system is integrable owing to a new connection with Schur measures. This leads to limit shape and KPZ-type fluctuation results. We also discuss a q-deformation of this system, and its mapping to a directed random polymer model.

Leandro Pimentel
Local Behavior of Airy Processes

Abstract: The Airy processes describe spatial fluctuations in wide range of growth models, where each particular Airy process arising in each case depends on the geometry of the initial profile. We show how the coupling method, developed in the last-passage percolation context, can be used to prove that several types of Airy processes have a continuous version, and behave locally like a Brownian motion.

Firas Rassoul-Agha
KPZ wandering exponent for random walk in i.i.d. dynamic Beta random environment

Abstract: We condition random walk in an i.i.d. dynamic Beta random environment to escape at an atypical velocity. The conditioned process converges to another random walk in random environment (RWRE). The new environment is a Doob transform of the original one by a harmonic function that is a Busemann type limit and solves a variational formula for the quenched large deviation rate function. Along the way we construct the stationary Beta polymer and prove fluctuation bounds for it. The Doob-conditioned RWRE is in duality with this polymer and, as a result, it has a KPZ wandering exponent of $2/3$.

Based on joint work with Marton Balazs and Timo Seppalainen.

Timo Seppalainen
Variational formulas and geodesics for percolation models

Abstract: We discuss recently found variational descriptions of the limit shape of first-passage percolation and some related ideas, such as a convex duality between the mean edge weight and the Euclidean length of the geodesic.

Based on joint work with Arjun Krishnan and Firas Rassoul-Agha.

Fabio Toninelli
Discrete interface dynamics and hydrodynamic limits

Abstract: Dimer models provide natural models of (2+1)-dimensional random discrete interfaces and of stochastic interface dynamics. I will discuss two examples of such dynamics, a reversible one and a driven one (growth process). In both cases we can prove the convergence of the stochastic interface evolution to a deterministic PDE after suitable (diffusive or hyperbolic respectively in the two cases) space-time rescaling.

Based on joint work with B. Laslier and M. Legras.

Ke Zhang

Hyperbolicity of minimizers and Random Hamilton-Jacobi equations

Abstract: We describe several results concerning a class of random Hamilton-Jacobi equations, the "one force, one solution" principle, regularity of stationary solutions, and speed of convergence to the stationary solutions. The fact underlying these results is the hyperbolicity of the global minimizing orbit, which allows us to apply theories from smooth dynamics.

Based on joint works with Renato Iturriaga and Konstantin Khanin.

Nikolaos Zygouras

High temperature limits of directed polymers with heavy tail disorder

Abstract: The directed polymer model at intermediate disorder regime was introduced by Alberts-Khanin-Quastel. Under the assumption of exponential moments, they proved that at inverse temperature $\beta n\gamma$, with $\gamma = 1/4$, the partition function, centered appropriately, converges in distribution to the solution of the one dimensional stochastic heat equation. This result was also conjectured to hold under the assumption of six moments. We will show that this conjecture is valid. Furthermore, we will fit this result into a more general phase diagram in (γ, α), with α the tail exponent of the disorder distribution, and we will describe which parts of this phase diagram are proven and which remain conjectural.

Based on joint work with Partha Dey.